interlayer friction thus reducing the liquid velocity. Moreover, the fact that points moving with opposite velocities are near each other, leads to a further reduction in the contribution of the motion of the liquid towards the moment of impulse of the solid-fluid system. Therefore the result is $P_{xx} \ll P_{zz}$.

The above discussion appears to be qualitatively applicable to more complex configurations of the closed tube. Two types of flow can exist in any such tube, one extending throughout the whole tube, and the other through a part of it. When a close vortex extending through the whole tube is present, then the moment L is larger than in its absence. For a plane tube the moment, and therefore the value of P_{ii} is largest when the angular acceleration is perpendicular to the plane of the tube.

The moment of forces acted upon the top by the liquid has a retarding influence on the angular acceleration. Therefore the liquid contained in the cavity of the top exerts a stabilizing influence [1, 4]. In the case when the diagonal components of P_{ii} are different from each other, an optimal orientation of the cavity exists for which the stabilization time is shortest [4]. For a torus this situation arises when the principal axes with the largest and smallest moment of inertia are parallel to the plane of the torus.

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UDC 538.4

ADIABATIC SHOCK CURVE IN MAGNETIZABLE NONCONDUCTING MEDIA

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We investigate the relationships at the discontinuities in magnetizable nonconducting media. The magnetic permeability is assumed to be an arbitrary function of the magnetic field and, generally speaking, different on each side of the discontinuity. We note that the contribution of the terms connected with the magnetizability towards the relations at the discontinuity is substantial also in the case when the values of permeability at both sides of the discontinuity are constant and different from each other. We show that the behavior of the adiabatic shock curve depends substantially on the sign of the difference in the values of the permeability ahead and behind the discontinuity.

The monograph [1] gives general expressions for the relations at the discontinuities in the mechanics of continuum, taking into account the electromagnetic field and the magnetization and polarization effects. The relations of the discontinuities are given in [2, 3] for the ferrohydrodynamic and electrohydrodynamic approximations.

In the case when the permeability μ depends only on the magnetic field, $\mu = \mu\,(H)$, in such a manner that, generally speaking, the function μ_2 behind the discontinuity is different from the function μ_1 ahead the discontinuity. The system of relations at the discontinuity has the form

$$\{\rho u_n\} = 0, \quad \{p_0\} + m^2\{v\} + \frac{1}{4\pi}\left\{\int_0^H \mu H dH\right\} = \frac{B_n^2}{4\pi}\left\{\frac{1}{\mu}\right\}, \quad m = \rho u_n \tag{1}$$

$$m\{\mathbf{u}_\tau\} = \frac{B_n}{4\pi}\{\mathbf{H}_\tau\}, \quad \mathbf{B} = \mu\mathbf{H}, \quad p_0 = f(\rho, T)$$

$$m\left\{\frac{v^2}{2}\right\} + \{w_0\} = 0, \quad \{\mathbf{H}_\tau\} = 0, \quad \{B_n\} = 0, \quad (\{a\} = a_2 - a_1)$$

where p_0 and w_0 denote the pressure and enthalpy in the absence of a magnetic field, ρ is the density, v is the specific volume, T is the temperature, $\mathbf{u}_\tau$ and u_n are the tangential and normal components of the velocity of the medium in a coordinate system in which the discontinuity is at rest. The subscripts 1 and 2 denote the parameters of the medium ahead and behind the discontinuity, respectively.

From the fourth and eighth equation of (1) it follows that the tangential velocity component does not undergo any discontinuity. The eighth and ninth equations of (1) give the magnetic field behind the discontinuity in terms of the specified field ahead the discontinuity. In a particular case when the values of permeability ahead and behind the discontinuity are constant but different from each other, we have $\mathbf{H}_{\tau 2} = \mathbf{H}_{\tau 1}$ and $H_{n2} = (\mu_1 / \mu_2)H_{n1}$. We shall call the shock wave the demagnetization (magnetization) wave if $\mu_2 < \mu_1 (\mu_2 > \mu_1)$. We note that the magnetic field behind the discontinuity can be determined independently of the other parameters, and in the case of an arbitrary dependence of the permeability on the magnetic field, the density and the temperature, the permeability becomes equal to unity behind the discontinuity when the latter is the demagnetization wave. Morover, we then have $\mathbf{H}_{\tau 2} = \mathbf{H}_{\tau 1}$ and $H_{n2} = \mu_1(\rho_1, T_1, H_1)H_{n1}$.

From the second, seventh and eighth equations of (1) follows:

$$w_{02} - w_{01} = \frac{v_1 + v_2}{2}(p_{02} - p_{01} + q) \tag{2}$$

$$m^2 = \frac{p_{02} - p_{01} + q}{v_1 - v_2}$$

$$q = \left\{\frac{1}{4\pi}\int_0^H \mu H dH - \frac{B_n^2}{4\pi\mu}\right\}$$

The quantity q is a known function of the parameters ahead the discontinuity. Its sign depends on the form

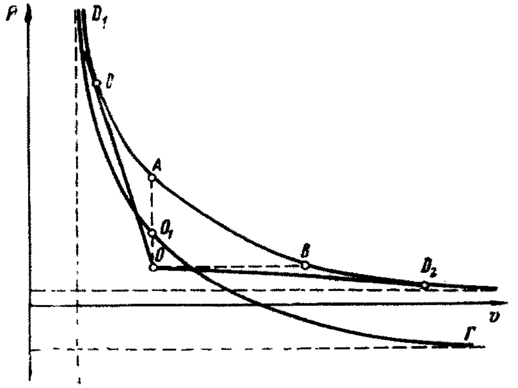

Fig. 1

of the function $\mu(H)$, e.g. in the case when the shock wave separates two different media each of which is characterized by its constant permeability, i.e. $\mu_1 = \text{const}$ and $\mu_2 = \text{const}$, we have $q = (\mu_2 - \mu_1)\,[H_{\tau 1}^2 + (\mu_2 / \mu_1) H_{n1}^2] / 8\pi$, so that for the magnetization waves $q > 0$, while for the demagnetization waves $q < 0$. In the case when $\mu(H)$ is a step function with a discontinuity at the point $H = H^*$, we have

$$q = (\mu_2 - \mu_1)\left[H_{\tau 1}^2 + \frac{\mu_2}{\mu_1} H_{n1}^2 + \frac{|\mu_2 - \mu_1|}{\mu_2 - \mu_1} H^{*2}\right]$$

and $q > 0$ for $\mu_2 > \mu_1$, as well as for $\mu_2 < \mu_1$.

We shall assume that the medium is described by the equation of state for an ideal gas and, that the enthalpy is given by $w_0 = \gamma p_0 v / (\gamma - 1)$. The equation (2) yields the following expression for the adiabatic shock curve in the pv-plane:

$$p_2 = \frac{\varkappa p_1 v_1 - p_1 v_2}{\varkappa v_2 - v_1} + \frac{q(v_1 + v_2)}{\varkappa v_2 - v_1}, \quad \varkappa = \frac{\gamma + 1}{\gamma - 1} \tag{3}$$

For the media with the permeability independent of temperature, the entropy of the medium becomes identical with the entropy s_0 in the absence of the magnetic field. It can be shown that for the adiabatic shock curve in magnetizable nonconducting media the relations $dm^2 / dp_2 = 0$, $ds_2 / dp_2 = 0$ and $u_{n2} = c_2$ (where c_2 is the speed of sound in the absence of a magnetic field), hold simultaneously just as in the conventional gas dynamics.

Let us construct an adiabatic shock curve for the case $q > 0$ (Fig. 1). We draw the Hugoniot curve through the point $O_1\,(v_1, p_1)$. The curve for magnetizable media described by Eq. (3) is a hyperbola with the asymptotes

$$\frac{v_2}{v_1} = \frac{1}{\varkappa}, \qquad \frac{p_2}{p_1} = -\frac{1}{\varkappa} + \frac{q}{\varkappa p_1}$$

which passes through the point $A\,(v_1, p_1 + q(\gamma - 1))$ and is situated above the curve Γ. From the second equation of (2) it follows that the tangent of the angle of inclination of the secant to the curve taken with the opposite sign and drawn from the point $O\,(v_1, p_1 - q)$ is equal to the square of the mass flux density. At the point D of contact of this secant to the curve, the normal component of velocity of the medium behind the discontinuity u_{n2} is equal to the speed of sound c_2, since at this point $dm^2 / dp_2 = 0$.

We shall show that $u_{n2} < c_2$ above the point D and $u_{n2} > c_2$ below it. Let us find the sign of the derivative

$$\frac{d(u_{n2}^2 - c_2^2)}{dv_2} = 2v_2 m^2 + v_2^2 \frac{dm_2^2}{dv_2} - \gamma v_2 \frac{dp_2}{dv_2} - \gamma p_2$$

Near the point D the expansions $m^2 = c_2^2 / v_2^2 + o(v_2)$ and $dm^2 / dp_2 = o(v_2)\, dp_2 / dv_2$ hold, and

$$d(u_{n2}^2 - c_2^2) / dv_2 = \gamma p_2 - (\gamma v_2 + o(v_2)) dp_2 / dv_2 + o(v_2)$$

In the case when $dp_2 / dv_2 < 0$ the derivative $d(u_{n2}^2 - c_2^2) / dp_2$ is positive near the point D, consequently the velocity $u_{n2} < c_2$ for $v_2 < v_D$ and $u_{n2} > c_2$ for $v_2 > v_D$. These inequalities hold at all points of the curve when $v_2 < v_1$.

The segment AB of the curve has no physical meaning since on it we have $m^2 < 0$. When $p_1 - q \leqslant 0$, the right-hand side branch of the adiabatic curve descends below the axis $p = 0$ and the pressure p_2 becomes negative, so that this part of the curve also has no physical meaning. On the segment AD_1 the velocity $u_{n1} > c_1$ since on this segment the angle of inclination of the secant projected from the point $O(v_1, p_1 - q)$ to the curve is always greater than the angle of inclination of the tangent to the Hugoniot curve at the point O_1.

Thus, on the segment DD_1 we have $u_{n2} < c_2$, $u_{n1} > c_1$, on DA the $u_{n2} > c_2$, $u_{n1} > c_1$ on BD_2 the $u_{n2} > c_2$, $u_{n1} < c_1$,

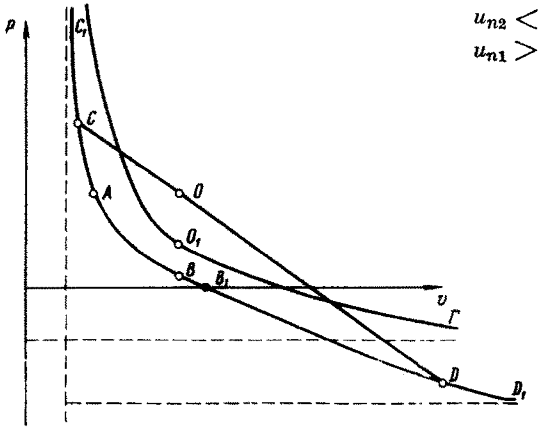

Fig. 2

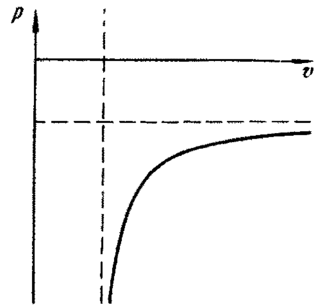

Fig. 3

and on D_2D_3 the $u_{n2} > c_2$, $u_{n1} < c_1$. For the shock waves in magnetizable nonconducting media the conditions of evolutionarity are satisfied only by the branch DD_1 unless supplementary equations not derivable from the laws of conservation are available at the surface of discontinuity.

Let us consider the case $q < 0$. The derivative

$$\frac{dp_2}{dv_2} = -|v_1 \frac{2\gamma\,[2p_1 + q(\gamma - 1)]}{(\gamma - 1)^2\,[\varkappa v_2 - v_1]^2}$$

can either be positive or negative, and this will significantly affect the form of the corresponding curves $p_2(v_2)$. When $p_1 = -q(\gamma - 1)/2$, the adiabatic curve degenerates into a straight line $p_2 = -p_1$. For the values of q satisfying the inequality

$$2p_1 + q(\gamma - 1) > 0$$

the curve has the form shown in Fig. 2. The curve $p_2(v_2)$ is situated below the Hugoniot curve. Beginning from the point B_1, the pressure p_2 is negative, while at $p_1 < -q(\gamma - 1)$ the whole branch BD_1 is situated below the axis $p = 0$ and has no physical meaning. The secant projected from the point $O(v_1, p_1 - q)$ again yields, as in the case $q > 0$, the mass flux across the surface of the discontinuity.

At the points C and D of intersection of the straight line passing through the point O and parallel to the tangent to the Hugoniot curve at the point O_1, the normal velocity ahead the discontinuity u_{n1} is equal to the speed of sound c_1. Above the point C we have $u_{n1} > c_1$, and below it we have $u_{n1} < c_1$. At the point C the following relations hold:

$$\frac{u_{n2}^2}{v_2^2} = \frac{c_1^2}{v_1^2} = \frac{\gamma p_1}{v_1} < \frac{\gamma p_2}{v_2} = \frac{c_2^2}{v_2^2}$$

From this it follows that $u_{n2} < c_2$ and since on this segment dm^2/dp_2 is never zero, the above inequality holds for the whole branch AC_1. Similarly, on the segment BD_1

we have $u_{n2} > c_2$. For the values of q satisfying the inequality (4) the conditions of evolutionarity hold on the segment CC_1 of the adiabatic curve. When the inequality $2p_1 + q(\gamma - 1) < 0$ holds, the function $p_2(v_2)$ increases monotonously from $-\infty$ at $v_2 = v_1 / \varkappa$ to $p_2 = -(p_1 - q) / \varkappa$ when v_2 tends to ∞ (Fig. 3). Thus, the whole of the curve $p_2(v_2)$ lies below the axis $p = 0$ and has no physical meaning.

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UDC 532.5

LONGITUDINAL FLOW PAST A SLENDER BODY OF REVOLUTION WITH A FREE BOUNDARY

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We investigate a longitudinal flow past an axisymmetric body in the case when a part of the streamlined surface is not known but instead, the distribution of the tangential velocities is specified. The flow is assumed irrotational, and the fluid ideal and incompressible. At the stagnation points the body surface may behave as a sphere, a cone or an edge. An integro-differential equation for determining the form of the free surface is derived for any arbitrarily specified velocity. In the case of a cavitation flow the method of the undetermined coefficients is used to solve the above equation. An analytic and graphical dependence of the cavitation number on the apex angle of cone and its relative length, is given. The theory is satisfactorily confirmed by experimental data.

1. Statement of the problem. Let a longitudinal irrotational stream of an ideal incompressible fluid flow past a slender axisymmetric body. The surface of this body is described by the equation $\rho = R(z)$, where

$$R(z) = \begin{cases} r_-(z) & -1 < z < b \\ r(z) \quad \text{as given by the condition} \quad v_\tau = v_\tau(z), & b < z < c \\ r_+(z) & c < z < 1 \end{cases} \qquad (1.1)$$

The segment (b, c) which is defined by the distribution of tangential velocities $v_\tau(z)$ is a free boundary, while the segments $(-1, b)$ and $(c, 1)$ are parts of the rigid boundary.

The problem is reduced to finding the equation of the free boundary $r(z)$. We assume